



GAM-036022 Seat No. _____

B. Sc. (Sem. VI) Examination

March / April - 2019

Mathematics : BSCC-606B

(Analysis - III)

Time : 3 Hours]

[Total Marks : 70

- Instructions :**
- (1) Do as directed.
 - (2) All questions are compulsory.
 - (3) Each question carry equal marks.

- 1 (a) In a metric space prove that every open sphere is an open set. 7

OR

Let X be a metric space then prove that

- (1) Any intersection of closed set is closed.
- (2) Finite union of closed sets is closed.

- (b) Let $X = R^2$ and $d : R^2 \times R^2 \rightarrow R$ be defined as 7

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \quad \text{where } x = (x_1, x_2)$$

$y = (y_1, y_2) \in R^2$ then prove that d is a metric on R^2 .

OR

Let X be a metric space, a subset F of X is closed iff its complement F^c is open.

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[Contd...

- 2 (a) Prove that if A is closed subset of a compact metric space (X, d) then (A, d) is compact. 7

OR

Let X and Y be metric space and f is a mapping of X into Y , then prove that f is continuous iff $f^{-1}(G)$ is open in X , whenever G is open in Y .

- (b) Prove that continuous image of a compact metric space is compact. 7

OR

Let X and Y be metric spaces and $f : X \rightarrow Y$ then prove that f is continuous at x_0 iff $f(x_n) \rightarrow f(x_0)$ whenever $x_n \rightarrow x_0$.

- 3 (a) Let $\{f_n\}$ be a sequence of continuous function on $[a, b]$ and suppose that $f_n \rightarrow f$ uniformly on $[a, b]$ then prove that 7

$$\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = \int_a^b \lim_{n \rightarrow \infty} f_n(x) dx.$$

OR

Suppose $\lim_{n \rightarrow \infty} f_n(x) = f(x) \forall x \in E$ and let

$M_n = \sup_{x \in E} |f_n(x) - f(x)|$ then $f_n \rightarrow f$ uniformly on E iff

$$\lim_{n \rightarrow \infty} M_n = 0.$$

(b) Let $f_n(x) = |x|^{1+\frac{1}{n}} \forall x \in [-1, 1]$ show that 7

(1) $f_n \in D[-1, 1]$

(2) $f_n(x) \Rightarrow f(x) = |x|$ uniformly on $[-1, 1]$

(3) f is not differentiable

OR

Prove that $f_n(x) = n^2 x^n (1-x) \forall x \in [0, 1]$ converges point wise but not uniformly to a function which is continuous in $[0, 1]$.

4 (a) State and prove Weierstrass approximation theorem. 7

OR

State and prove Abel's limit theorem.

(b) For every $x \in R$ and $n \geq \theta$ prove that 7

(1)
$$\sum_{k=\theta}^n (nx - k)^2 \binom{n}{k} x^k (1-x)^{n-k} = nx(1-x)$$

(2)
$$\sum_{k=\theta}^n \binom{n}{k} x^k (1-x)^{n-k} = 1$$

OR

Show that the function

$$f(x) = \left. \begin{array}{l} \frac{1}{e^{x^2}} \quad x \neq 0 \\ 0 \quad x = 0 \end{array} \right\} \text{ has derivatives of all orders at all}$$

$x \neq 0$ but doesnot have a Taylor's theorem.

5 Answer in short : (any seven)

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- (1) Define : Open set and closed set.
 - (2) Define : Convergence and completeness.
 - (3) Define : Continuity and compactness.
 - (4) Define : Pointwise and uniformly convergence.
 - (5) Define : A derive set. Give one example.
 - (6) Define : A connected set. Give an example of connected set.
 - (7) If $f_n(x) = \frac{1}{1+nx}$ ($x \geq 0$) then check continuity of $f_n(x)$.
 - (8) Define : Metric space.
 - (9) Explain Taylor's series.
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